



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

May/June 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.

1 Find the solution of the differential equation

$$\frac{dy}{dx} + 5y = e^{-7x}$$

for which $y = 0$ when $x = 0$. Give your answer in the form $y = f(x)$.

[6]

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2 It is given that $y = 2^x$.

(a) By differentiating $\ln y$ with respect to x , show that $\frac{dy}{dx} = 2^x \ln 2$. [3]

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(b) Write down $\frac{d^2y}{dx^2}$. [1]

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(c) Hence find the first three terms in the Maclaurin's series for 2^x . [3]

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- 3 (a) Find the roots of the equation $z^3 = -1 - i$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $0 \leq \theta < 2\pi$. [5]

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Let $w = z_1^{3k} + z_2^{3k} + z_3^{3k}$, where k is a positive integer and z_1, z_2, z_3 are the roots of $z^3 = -1 - i$.

- (b) Express w in the form $Re^{i\alpha}$, where $R > 0$, giving R and α in terms of k . [3]

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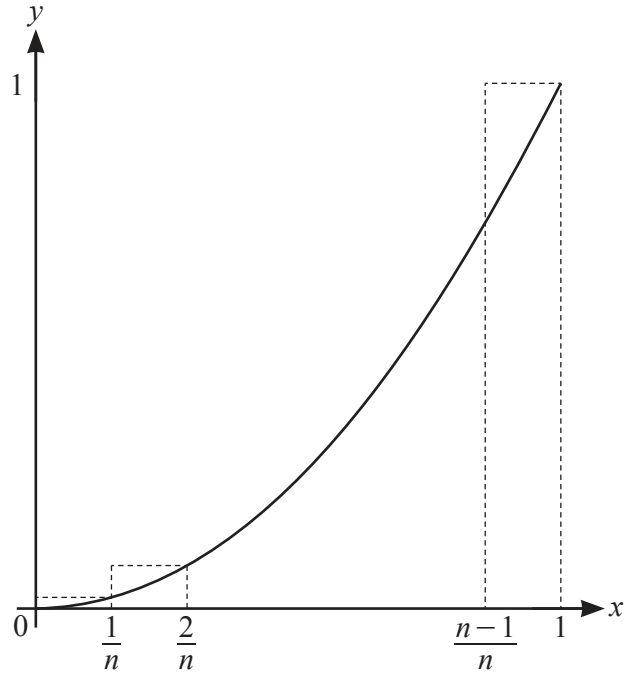
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The diagram shows the curve with equation $y = x^2$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that

$$\int_0^1 x^2 dx < \frac{2n^2 + 3n + 1}{6n^2}. \tag{4}$$

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(b) Use a similar method to find, in terms of n , a lower bound for $\int_0^1 x^2 dx$. [4]

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5 The curves $C_1 : y = \cosh x$ and $C_2 : y = \sinh 2x$ intersect at the point where $x = a$.

(a) Find the exact value of a , giving your answer in logarithmic form.

[4]

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(b) Sketch C_1 and C_2 on the same diagram.

[2]

6 The integral I_n , where n is an integer, is defined by $I_n = \int_0^{\frac{1}{2}} (1-x^2)^{-\frac{1}{2}n} dx$.

(a) Find the exact value of I_1 . [2]

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(b) By considering $\frac{d}{dx} (x(1-x^2)^{-\frac{1}{2}n})$, or otherwise, show that

$$nI_{n+2} = 2^{n-1} 3^{-\frac{1}{2}n} + (n-1)I_n. [5]$$

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7 It is given that $x = t^3y$ and

$$t^3 \frac{d^2y}{dt^2} + (4t^3 + 6t^2) \frac{dy}{dt} + (13t^3 + 12t^2 + 6t)y = 6te^{\frac{1}{2}t}.$$

(a) Show that

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 6te^{\frac{1}{2}t}. \tag{4}$$

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(b) Find the general solution for y in terms of t . [7]

A series of horizontal dotted lines for writing the answer.

- 8 (a) Find the values of a for which the system of equations

$$3x + y + z = 0,$$

$$ax + 6y - z = 0,$$

$$ay - 2z = 0,$$

does not have a unique solution.

[3]

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The matrix A is given by

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 6 & -1 \\ 0 & 0 & -2 \end{pmatrix}.$$

- (b) Use the characteristic equation of A to find the inverse of A^2 .

[4]

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Additional Page

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